DECAY CONSTANTS OF THE PSEUDOSCALAR MESONS IN THE FRAMEWORK OF THE COUPLED SCHWINGER-DYSON EQUATION AND BETHE-SALPETER EQUATION

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In this article, we investigate the structures of the pseudoscalar mesons (π, K, D, D_s, B and B_s) in the framework of the coupled rainbow Schwinger-Dyson equation and ladder Bethe-Salpeter equation with the confining effective potential (infrared modified flat bottom potential). The Schwinger-Dyson functions for the u, d and s quarks are greatly renormalized at small momentum region and the curves are steep at about $q^2 = 1GeV^2$ which indicates an explicitly dynamical symmetry breaking. The Euclidean time fourier transformed quark propagators have no mass poles in the time-like region which naturally implements confinement. As for the c and b quarks, the current masses are very large, the renormalization are more tender, however, mass poles in the time-like region are also absent. The Bethe-Salpeter wavefunctions for those mesons have the same type (Gaussian type) momentum dependence and center around small momentum which indicate that the bound states exist in the infrared region. The decay constants for those pseudoscalar mesons are compatible with the values of experimental extractions and theoretical calculations, such as lattice simulations and QCD sum rules.

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I. INTRODUCTION

The low energy nonperturbative properties of quantum chromodynamics (QCD) put forward a great challenge for physicists as the strong $SU(3)$ gauge coupling at small momentum region destroys the perturbative expansion approach, which is popular in theoretical physics. The physicists propose many original approaches to deal with the long distance properties of QCD, such as Chiral perturbation theory [1], heavy quark effective theory [2], QCD sum rules [3], lattice QCD [4], perturbative QCD [5], coupled Schwinger-Dyson equation (SDE) and Bethe-Salpeter equation (BSE) method [6], etc. All of those approaches have both outstanding advantages and obvious shortcomings in one or other ways. The coupled rainbow SDE and ladder BSE have given a lot of successful descriptions of the long distance properties of the low energy QCD and the QCD vacuum (for example, Refs. [7–10] , for recent reviews

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one can see Refs. [11,12]). The SDE can naturally embody the dynamical symmetry breaking and confinement which are two crucial features of QCD, although they correspond to two very different energy scales [13,14]. On the other hand, the BSE is a conventional approach in dealing with the two body relativistic bound state problems [15]. From the solutions of the BSE, we can obtain useful information about the under-structure of the mesons and obtain powerful tests for the quark theory. However, the obviously drawback may be the model dependent kernels for the gluon two point Green's function and the truncations for the coupled divergent SDE and BSE series in one or the other ways [21]. Many analytical and numerical calculations indicate that the coupled rainbow SDE and ladder BSE with phenomenological potential models can give model independent results and satisfactory values [6–12]. The usually used effective potential models are confining Dirac δ function potential, Gaussian distribution potential and flat bottom potential (FBP) [11,12,16–18]. The FBP is a sum of Yukawa potentials, which not only satisfies gauge invariance, chiral invariance and fully relativistic covariance, but also suppresses the singular point which the Yukawa potential has. It works well in understanding the dynamical chiral symmetry braking, confinement and the QCD vacuum as well as the meson structures, such as electromagnetic form factors, radius, decay constants [19–21].

The decay constants of the pseudoscalar mesons play an important role in modern physics with the assumption of current-meson duality. The precise knowledge of the those values (especially the f_D, f_{D_s}, f_B and f_{B_s}) will provide great improvements in our understanding of various processes convolving the D, D_s, B and B_s mesons decays. At present, it is a great challenge to extract the values of the B and B_s mesons decay constants f_B and f_{B_s} from the experimental data while the experimental values for the D and D_s mesons decay constants f_D and f_{D_s} are plagued with large uncertainties. So it is interesting to combine those successful potentials within the framework of coupled SDE and BSE to calculate the decay constants of the pseudoscalar mesons such as π, K, D, D_s, B, and B_s while we prefer the detailed studies of the quarkonium to another article. For previous studies about the electroweak decays of the pseudoscalar mesons with the SDE and BSE, one can consult Refs. [6–12]. In this article, we use an infrared modified flat-bottom potential (IMFBP) which takes the advantages of both the Gaussian distribution potential and the FBP to calculate the decay constants of those pseudoscalar mesons. The present article is a continuation of our previous work [21].

The article is arranged as follows: we introduce the infrared modified flat bottom potential in section II; in section III, IV and V, we solve the rainbow Schwinger-Dyson equation and ladder Bethe-Salpeter equation, explore the analyticity of the quark propagators, investigate the dynamical symmetry breaking and confinement, finally obtain

the decay constants for those pseudoscalar mesons; section VI is reserved for conclusion.

II. INFRARED MODIFIED FLAT BOTTOM POTENTIAL

The present techniques in QCD calculation can not give satisfactory large r behavior for the gluon two point Green's function to implement the linear potential confinement mechanism ‡, in practical manipulation, the phenomenological effective potential models always do the work. As in our previous work [21], we use a gaussian distribution function to represent the infrared behavior of the gluon two point Green's function,

$$4\pi G_1(k^2) = 3\pi^2 \frac{\varpi^2}{\Delta^2} e^{-\frac{k^2}{\Delta}}, \tag{1}$$

which determines the quark-antiquark interaction through a strength parameter ϖ and a ranger parameter Δ. This form is inspired by the δ function potential (in other words the infrared dominated potential) used in Refs. [16,17], which it approaches in the limit $\Delta \to 0$. For the intermediate momentum, we take the FBP as the best approximation and neglect the large momentum contributions from the perturbative QCD calculations as the coupling constant at high energy is very small. The FBP is a sum of Yukawa potentials which is an analogy to the exchange of a series of particles and ghosts with different masses (Euclidean Form),

$$G_2(k^2) = \sum_{j=0}^{n} \frac{a_j}{k^2 + (N + j\rho)^2}, \tag{2}$$

where N stands for the minimum value of the masses, ρ is their mass difference, and a_j is their relative coupling constant. Due to the particular condition we take for the FBP, there is no divergence in solving the SDE. In its three dimensional form, the FBP takes the following form:

$$V(r) = -\sum_{j=0}^{n} a_j \frac{\mathrm{e}^{-(N+j\rho)r}}{r}. \tag{3}$$

In order to suppress the singular point at $r = 0$, we take the following conditions:

$$\begin{gathered} V(0) = constant, \\ \frac{dV(0)}{dr} = \frac{d^2V(0)}{dr^2} = \cdots = \frac{d^nV(0)}{dr^n} = 0. \end{gathered} \tag{4}$$

So we can determine a_j by solve the following equations, inferred from the flat bottom condition Eq.(4),

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‡Here we correct a writing error in our article [21] i.e. change "small r" to "large r"

$$\sum_{j=0}^{n} a_j = 0,$$
$$\sum_{j=0}^{n} a_j (N + j\rho) = V(0),$$
$$\sum_{j=0}^{n} a_j (N + j\rho)^2 = 0,$$
$$\cdots$$
$$\sum_{j=0}^{n} a_j (N + j\rho)^n = 0. \tag{5}$$

As in previous literature [18–21], n is set to be 9. The phenomenological effective potential (infrared modified flat bottom potential) can be taken as $G(k^2) = G_1(k^2) + G_2(k^2)$.

III. SCHWINGER-DYSON EQUATION

The Schwinger-Dyson equation can provide a natural framework for investigating the nonperturbative properties of the quark and gluon Green's functions. By studying the evolution behavior and analytic structure of the dressed quark propagators, we can obtain valuable information about the dynamical symmetry breaking phenomenon and confinement. In the following, we write down the SDE for the quark propagator,

$$S^{-1}(p) = i\gamma \cdot p + m + \frac{16\pi}{3} \int \frac{d^4k}{(2\pi)^4} \Gamma_\mu S(k) \gamma_\nu G_{\mu\nu}(k - p), \tag{6}$$

where

$$S^{-1}(p) = iA(p^2)\gamma \cdot p + B(p^2) \equiv A(p^2)[i\gamma \cdot p + m(p^2)], \tag{7}$$
$$G_{\mu\nu}(k) = (\delta_{\mu\nu} - \frac{k_\mu k_\nu}{k^2})G(k^2), \tag{8}$$

and m stands for an explicit quark mass-breaking term. In this article, we take the rainbow approximation $\Gamma_\mu = \gamma_\mu$. With the explicit small mass term for the u and d quarks (comparing with the u and d quarks, the mass term for the s, c and b quarks is large), we can preclude the zero solution for the $B(p)$ and in fact there indeed exists a small bare current quark mass. In this article, we take Landau gauge. This dressing comprises the notation of constituent quark by providing a mass $m(p^2) = B(p^2)/A(p^2)$, for the u, d and s quarks, which is corresponding to the dynamical symmetry breaking. Because the form of the gluon propagator $G(p)$ in the infrared region can not be exactly inferred from the $SU(3)$ color gauge theory, one often uses model dependent forms as input parameters in the previous studies

of the rainbow SDE [6,11,12,18–22], in this article we use the infrared modified FBP to substitute for the gluon propagator.

In this article, we assume that a Wick rotation to Euclidean variables is allowed, and perform a rotation analytically continuing p and k into the Euclidean region where they can be denoted by $\bar{p}$ and $\bar{k}$, respectively. Alternatively, one can derive the SDE from the Euclidean path-integral formulation of the theory, thus avoiding possible difficulties in performing the Wick rotation [23] . As far as only numerical results are concerned, the two procedures are equal. In fact, the analytical structures of quark propagators have interesting information about confinement, we will make detailed discussion about the u, d, s, c and b quarks propagators respectively in section V.

The Euclidean rainbow SDE can be projected into two coupled integral equations for $A(\bar{p}^2)$ and $B(\bar{p}^2)$, the explicit expressions for those equations can be found in Ref. [19,20]. For simplicity, we ignore the bar on p and k in the following notations.

IV. BETHE-SALPETER EQUATION

The BSE is a conventional approach in dealing with the two body relativistic bound state problems [15]. The quark theory of the mesons suppose that the mesons are quark and antiquark bound states. The precise knowledge about the quark structures of the mesons will result in better understanding of their properties. In the following, we write down the ladder BSE for the pseudoscalar mesons,

$$S_+^{-1}(q+\xi P)\chi(q,P)S_-^{-1}(q-(1-\xi)P) = \frac{16\pi}{3}\int \frac{d^4k}{(2\pi)^4}\gamma_\mu \chi(k,P)\gamma_\nu G_{\mu\nu}(q-k), \tag{9}$$

where $S(q)$ is the quark propagator, $G_{\mu\nu}(k)$ is the gluon propagator, P_μ is the four-momentum of the center of mass of the pseudoscalar mesons, q_μ is the relative four-momentum between the quark and antiquark in the pseudoscalar mesons, γ_μ is the bare vertex of quark-gluon, ξ is the center of mass parameter which can be chosen to between 0 and 1, and $\chi(q,P)$ is the Bethe-Salpeter wavefunction (BSW) of the bound state.

We can perform the Wick rotation analytically and continue q and k into the Euclidean region [§], the Euclidean pseudoscalar BSW $\chi(q,P)$ can be expanded in lorentz-invariant functions and $SO(4)$ eigenfunctions, Tchebychev polynomials $T_n^{\frac{1}{2}}(\cos\theta)$. In the lowest order approximation, the BSW $\chi(q,P)$ takes the following form,

[§]To avoid possible difficulties in performing the Wick rotation, one can derive the BSE from the Euclidean path-integral formulation of the theory. As far as only numerical results are concerned, the two procedures are equal.

$$\chi(q,P)=\gamma_5\left[iF_1^0(q,P)+\gamma\cdot PF_2^0(q,P)+\gamma\cdot qq\cdot PF_3^1(q,P)+i[\gamma\cdot q,\gamma\cdot P]F_4^0(q,P)\right]. \quad (10)$$

It is important to translate the wavefunctions F_i^n into the same mass dimension to facilitate the calculations in solving the BSE,

$$F_1^0\to F_1^0,\ F_2^0\to\Lambda^1F_2^0,\ F_3^1\to\Lambda^3F_3^1,\ F_4^0\to\Lambda^2F_4^0,\ q\to q/\Lambda,\ P\to P/\Lambda\,, \quad (11)$$

where Λ is some quantity of the dimension of mass. Then the ladder BSE can be projected into the following four coupled integral equations,

$$\begin{aligned}
&H(1,1)F_1^0(q,P)+H(1,2)F_2^0(q,P)+H(1,3)F_3^1(q,P)+H(1,4)F_4^0(q,P)=\int_0^\infty k^3dk\int_0^\pi\sin^2\theta K(1,1),\\
&H(2,1)F_1^0(q,P)+H(2,2)F_2^0(q,P)+H(2,3)F_3^1(q,P)+H(2,4)F_4^0(q,P)=\int_0^\infty k^3dk\int_0^\pi\sin^2\theta(K(2,2)+K(2,3)),\\
&H(3,1)F_1^0(q,P)+H(3,2)F_2^0(q,P)+H(3,3)F_3^1(q,P)+H(3,4)F_4^0(q,P)=\int_0^\infty k^3dk\int_0^\pi\sin^2\theta(K(3,2)+K(3,3)),\\
&H(4,1)F_1^0(q,P)+H(4,2)F_2^0(q,P)+H(4,3)F_3^1(q,P)+H(4,4)F_4^0(q,P)=\int_0^\infty k^3dk\int_0^\pi\sin^2\theta K(4,4),
\end{aligned} \quad (12)$$

the expressions of the $H(i,j)$ and $K(i,j)$ are cumbersome and neglected here.

Here we will give some explanations for the expressions of $H(i,j)$. The $H(i,j)$'s are functions of the quark's Schwinger-Dyson functions (SDF) $A(q^2+\xi^2P^2+\xi q\cdot P)$, $B(q^2+\xi^2P^2+\xi q\cdot P)$, $A(q^2+(1-\xi)^2P^2-(1-\xi)q\cdot P)$ and $B(q^2+(1-\xi)^2P^2-(1-\xi)q\cdot P)$. The relative four-momentum q is a quantity in the Euclidean space-time while the center of mass four-momentum P is a quantity in the Minkowski space-time. The present theoretical techniques can not solve the SDE in the Minkowski space-time, we have to expand A and B in terms of Taylor series of $q\cdot P$, for example,

$$A(q^2+\xi^2P^2+\xi q\cdot P)=A(q^2+\xi^2P^2)+A(q^2+\xi^2P^2)'\xi q\cdot P+\cdots.$$

The other problem is that we can not solve the SDE in the time-like region as the two point gluon Green's function can not be exactly inferred from the $SU(3)$ color gauge theory even in the low energy space-like region. In practical manipulations, we can extrapolate the values of A and B from the space-like region smoothly to the time-like region with suitable polynomial functions. To avoid possible violation with confinement in sense of the appearance of pole masses $q^2=-m(q^2)$ in the time-like region, we must be care in choosing the polynomial functions [17].

Finally we write down the normalization condition for the BSW,

$$\int\frac{d^4q}{(2\pi)^4}\{\bar{\chi}\frac{\partial S^{-1}(q+\xi P)}{\partial P_\mu}\chi(q,P)S^{-1}(q-(1-\xi)P)+\bar{\chi}S^{-1}(q+\xi P)\chi(q,P)\frac{\partial S^{-1}(q-(1-\xi)P)}{\partial P_\mu}\}=2P_\mu, \quad (13)$$

where $\bar{\chi} = \gamma_4 \chi^+ \gamma_4$. We can substitute the expressions of the BSWs and SDFs into the above equation and obtain the analytical result, however, the expressions are cumbersome and neglected here.

V. COUPLED RAINBOW SD EQUATION AND LADDER BS EQUATION AND THE DECAY CONSTANTS

In this section, we explore the coupled equations of the rainbow SDE and ladder BSE for the pseudoscalar mesons. In solving those equations numerically, the simultaneous iterations converge quickly to an unique value independent of the choice of initial wavefunctions. The final results for the SDFs and BSWs are plotted as functions of the square momentum q^2.

The quark-gluon vertex can be dressed through the solutions of the Ward-Takahashi identity or Salvnov-Taylor identity and taken to be the Ball-Chiu vertex and Curtis-Pennington vertex [24,25]. Although it is possible to solve the SDE with the dressed vertex, our analytical results indicate that the expressions for the BSEs with the dressed vertex are cumbersome and not suitable for numerical iterations **.

In order to demonstrate the confinement of quarks, we have to study the analyticity of SDFs for the u, d, s, c and b quarks, and prove that there no mass poles on the real timelike q^2 axial. It is necessary to perform an analytical continuation of the dressed quark propagators from the Euclidean space into the Minkowski space $q_4 \to iq_0$. However, we have no knowledge about the singularity structure of quark propagators in the whole complex plane as our solutions are obtained in the Euclidean regions. We can take an alternative safe procedure, stay completely in the Euclidean space and take the Fourier transform with respect to the Euclidean time T for the scalar part (S_s) of the quark propagator [6,11,26],

$$\begin{aligned} S_s^*(T) &= \int_{-\infty}^{+\infty} \frac{dq_4}{2\pi} e^{iq_4 T} S_s, \\ &= \int_{-\infty}^{+\infty} \frac{dq_4}{2\pi} e^{iq_4 T} \frac{B(q^2)}{q^2 A^2(q^2) + B^2(q^2)} |_{\overrightarrow{q}=0}, \end{aligned} \tag{14}$$

where the 3-vector part of q is set to zero. If S(q) has a mass pole at $q^2 = -m^2(q^2)$ in the real timelike region, the Fourier transformed $S_s^*(T)$ would fall off as e^{-mT} for large T or $\log S_s^* = -mT$.

In our numerical calculations, for small T, the values of S_s^* are positive and decrease rapidly to zero and beyond with the increase of T, which are compatible with the result from lattice simulations [27] ; for large T, the values

**This observation is based on the authors's work in USTC.

of S_s^* are negative, except occasionally a very small fraction positive values. We can express S_s^* as $|S_s^*|e^{in\pi}$, n is an odd integer, $\log S_s^* = \log|S_s^*| + in\pi$. If we neglect the imaginary part, we find that when the Euclidean time T is large, there indeed exists a crudely approximated (almost flat) linear function with about zero slope for all the u, d (in isospin symmetry limit, the u and d quarks are equal), s, c and b quarks with respect to the variable T, which is shown in Fig.1. Here the word 'crudely' should be understand in the linearly fitted sense, to be exact, there is no linear function. However, such crudely fitted linear functions are hard to acquire physical explanation and the negative values for S_s^* indicate an explicit violation of the axiom of reflection positivity [28], in other words, the quarks are not physical observable i.e. confinement.

From Fig.2, we can see that for the u, d (in isospin symmetry limit) and s quarks, the SDFs are greatly renormalized at small momentum region and the curves are steep at about $q^2 = 1GeV^2$ which indicates an explicit dynamical chiral symmetry breaking, while at large q^2, they take asymptotic behavior. As for the c and b quarks, shown in Fig.3, the current masses are very large, the renormalization is more tender, however, mass poles in the time-like region are also absent, which can be seen from Fig.1. At zero momentum, $m_u(0) = 688MeV$, $m_d(0) = 688MeV$, $m_s(0) = 882MeV$, $m_c(0) = 1823MeV$ and $m_b(0) = 4960MeV$, which are compatible with the Euclidean constituent quark masses defined by $m^2(q^2) = q^2$ in the literature. From the plotted BSWs (see Fig.4 as an example), we can see that the BSWs for pseudoscalar mesons have the same type (Gaussian type) momentum dependence while the quantitative values are different from each other. The gaussian type BS wavefunctions which center around small momentum indicate that the bound states exist in the infrared region. Finally we obtain the values for the decay constants of those pseudoscalar mesons which are defined by

$$\begin{aligned} if_\pi P_\mu &= \langle 0|\bar{q}\gamma_\mu\gamma_5 q|\pi(P)\rangle, \\ &= \sqrt{N_c}\int Tr\left[\gamma_\mu\gamma_5\chi(k,P)\frac{d^4k}{(2\pi)^4}\right], \end{aligned} \tag{15}$$

here we use π to represent the pseudoscalar mesons,

$$f_\pi = 127MeV;\ \ f_K = 157MeV;\ \ f_D = 238MeV;\ \ f_{D_s} = 241MeV;\ \ f_B = 193MeV;\ \ f_{B_s} = 195MeV, \tag{16}$$

which are compatible with the results from the experimental extractions, lattice simulations and QCD sum rules calculations, $f_\pi = 130MeV(Exp)$, $f_K = 160MeV(Exp)$, $f_D = 300^{+180+80}_{-150-40}MeV(Exp)$, $f_{D_s} = 285\pm19\pm40MeV(Exp)$, $f_D \approx 160-230MeV(Latt, sumrule)$, $f_{D_s} \approx 210-260MeV(Latt, sumrule)$, $f_B \approx 150-230MeV(Latt, sumrule)$

and $f_{B_s} \approx 190 - 260MeV(Latt, sumrule)$ [29–31]. The values from the lattice simulations and QCD sum rules calculations vary in a broad range, here we make an approximation. In calculation, the input parameters are $N = 1.0\Lambda$, $V(0) = -11.0\Lambda$, $\rho = 5.0\Lambda$, $m_u = m_d = 6MeV$, $m_s = 150MeV$, $m_c = 1250MeV$, $m_b = 4700MeV$, $\Lambda = 200MeV$, $\varpi = 1.6GeV$ and $\Delta = 0.04GeV^2$, the masses of the pseudoscalar mesons are taken as input parameters [21].

VI. CONCLUSION

In this article, we investigate the under-structures of the pseudoscalar mesons (π, K, D, D_s, B and B_s) in the framework of the coupled rainbow SD equation and ladder BS equation with the confining effective potential (infrared modified flat bottom potential). Although the quark-gluon vertex can be dressed through the solutions of the Ward-Takahashi identity or Salvnov-Taylor identity and taken to be the Ball-Chiu vertex and Curtis-Pennington vertex, a consistently numerical manipulation is unpractical, we take bare approximation for the vertex $\Gamma_\mu = \gamma_\mu$. After we solve the coupled rainbow SDE and ladder BSE numerically, we obtain the SDFs and BSWs for the pseudoscalar mesons. The SDFs for the u, d and s quarks are greatly renormalized at small momentum region and the curves are steep at about $q^2 = 1GeV^2$ which indicates an explicitly dynamical chiral symmetry breaking. After we take the Euclidean time fourier transformation about the quark propagators, we can find that there are no mass poles in the time-like region, and obtain satisfactory results about confinement. As for the c and b quarks, the current masses are very large, the renormalization is more tender, however, mass poles in the time-like region are also absent. The BSWs for the pseudoscalar mesons have the same type (Gaussian type) momentum dependence while the quantitative values are different from each other. The gaussian type BS wavefunctions which center around small momentum indicate that the bound states exist in the infrared region. Our numerical results for the values of the decay constants of the pseudoscalar are compatible with the corresponding ones obtained from the experimental extractions and other theoretical calculations, such as lattice simulations and QCD sum rules. Once the satisfactory SDFs and BSWs for the pseudoscalar mesons are known, we can use them to investigate a lot of important quantities in the B meson decays, such as $B-\pi$, $B-K$, $B-D$, $B-\rho$ former factors, Isgur-Wise functions, strong coupling constants, etc.

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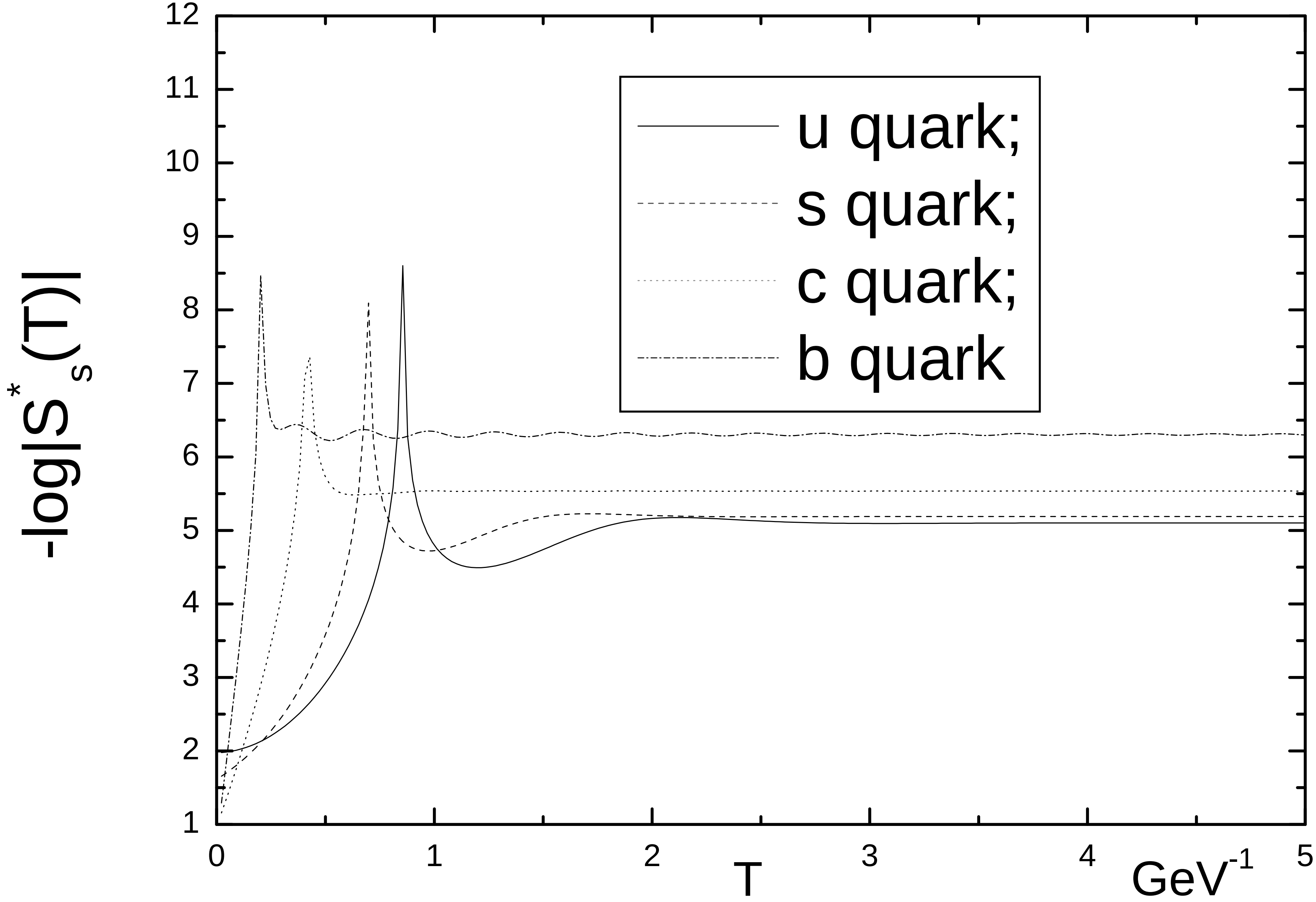
u quark;
s quark;
c quark;
b quark
-log|S*s(T)|
T
GeV-1
0
1
2
3
4
5
1
2
3
4
5
6
7
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9
10
11
12

Fig.1 $-\log|S^*_s(T)|$

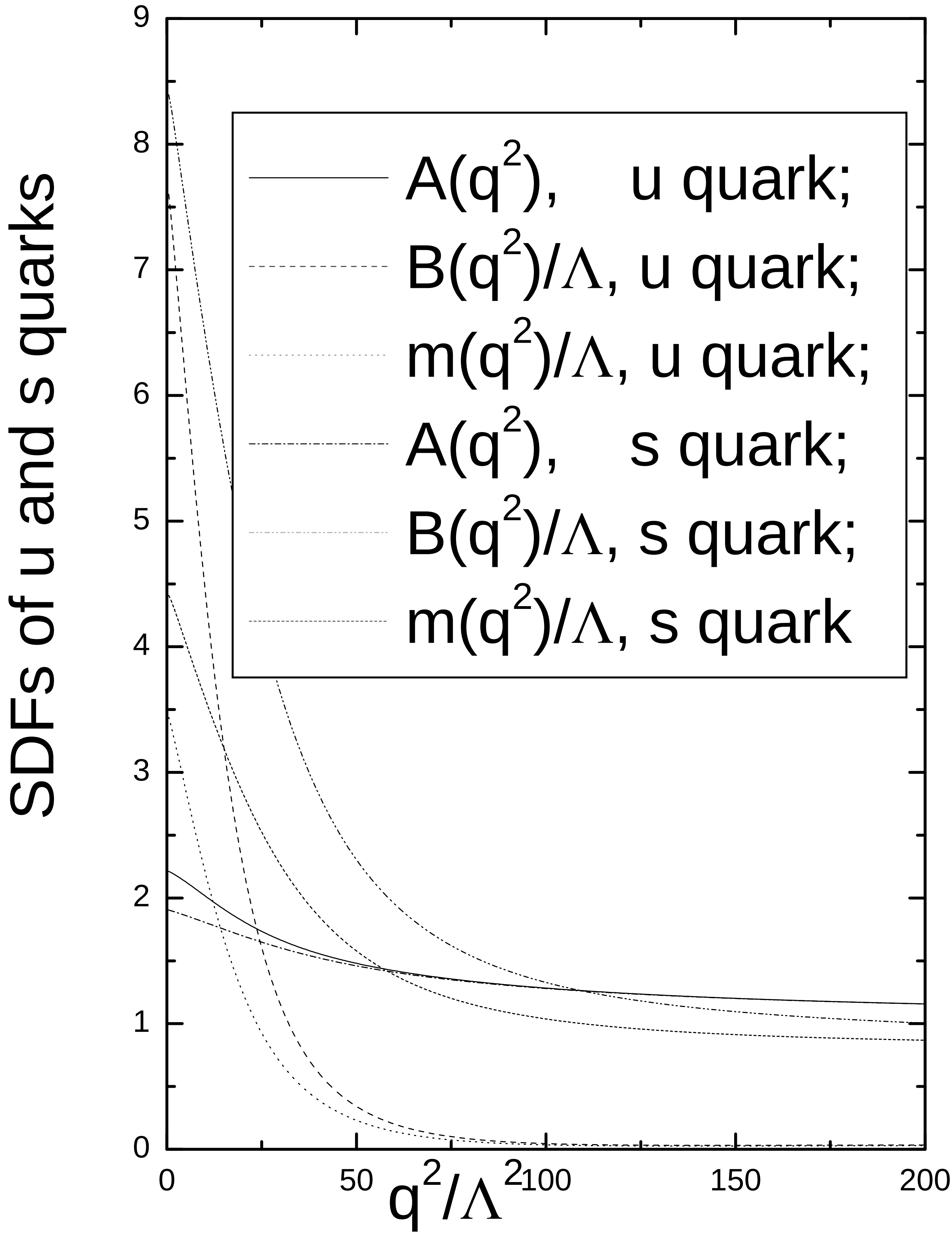

Fig.2 SDFs of u and s quarks

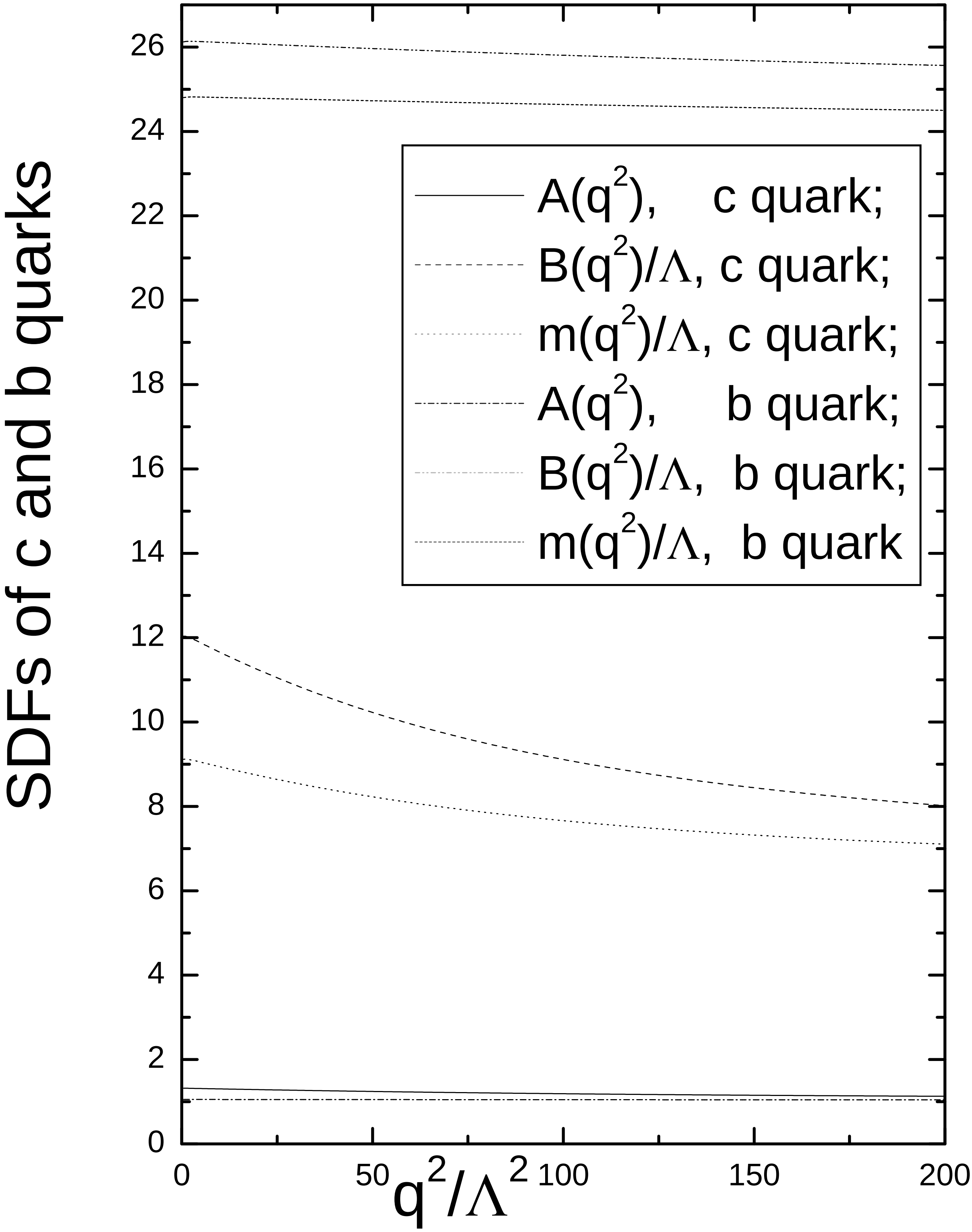

Fig.3 SDFs of c and b quarks

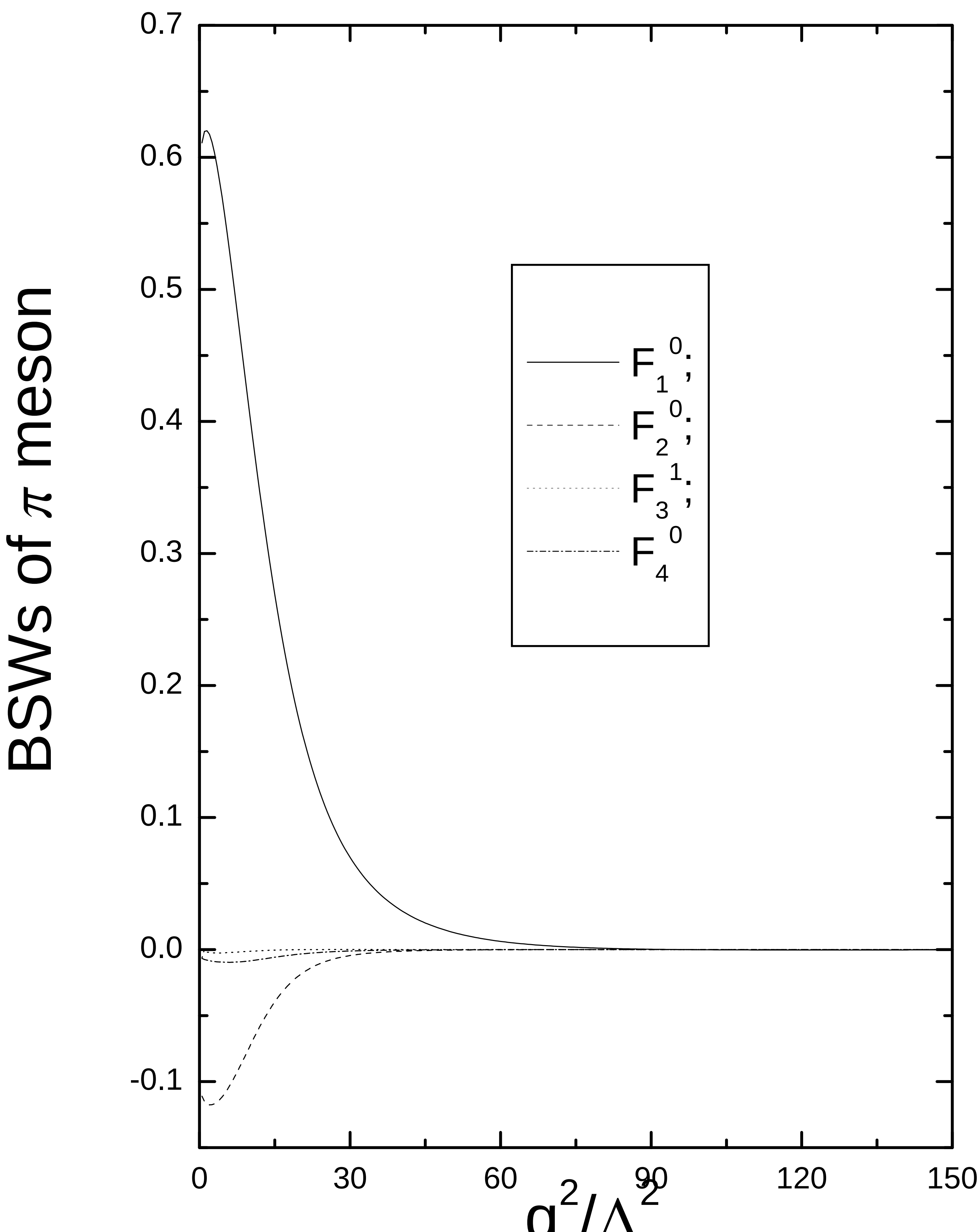

Fig.4 BSWs of π meson

FIG. 1. Fig.1 $-Log|S_s^*(T)|$.

FIG. 2. Fig.2 SDFs of u and s quarks.

FIG. 3. Fig.3 SDFs of c and b quarks.

FIG. 4. Fig.4 BSWs of π meson.